ISI – Bangalore Center – B Math - Physics III –Back Exam
Date: 1 Jan 2018. Duration of Exam: 3 hours
Total marks: 80

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Q1. [Total Marks: 4+6+8=18]

a.) Explain why the Electric field $\vec{E} = \alpha(xy\vec{i} + yz\vec{j} + zx\vec{k})$ where α is a constant cannot be produced by any static electric charge configuration.

b.) A charge *q* is placed *d* distance away from a infinite conducting plane kept at zero potential. What is the force felt by the charge? What is the work required to bring the *q* from infinity to its location.

c.) A sphere of radius *R* carries a charge density $\rho(\vec{r}) = kr$ where *k* is constant. Find the electric field at all points and show that the energy configuration of the system is given by $\frac{\pi k^2 R^7}{7\varepsilon_o}$.

Q2. [Total Marks: 4+4+6=14]

2a.) Explain why the electric field inside a conductor is zero in an electrostatic system..

2b.) Using the above show that the surface of a conductor must be an equipotential.

2c.) A spherical conducting body has a cavity of an arbitrary shape inside it. A charge *q* is kept in the cavity in such a way that it does not touch the conducting surface. Using the uniqueness of solution of Laplace equation with specified boundary conditions, determine the electric field at any point outside the conducting surface. Clearly explain why your answer does not depend on exact location or shape of the cavity within the conducting body.

Q3. [Total Marks:2+8+4 =14]

3a.) State the Biot Savart law for magnetic field produced by current I.

3b.) Using the Biot Savart law, show the magnitude due a wire segment carrying current I at a point which is located at a distance s from the wire (as shown in the figure) is given by $\frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1)$. What is its direction?

3c.) Find the magnetic field at the center of a square loop of side $2a$ which carries a steady current I .

Q4. [Total Marks: 4+8+2=14]

Suppose a parallel plate circular capacitor of radius a is being charged with a steady current I. Assume that the charge deposited on the capacitor plates is immediately uniformly distributed over the capacitor plates. Also assume the capacitor plates are close enough so that the electric filed can be taken as perpendicular to the surface.

4a.) Show that the electric field as a function of time is given by $\frac{It}{\pi\varepsilon_o a^2}\hat{z}$.

4b.) Show that the magnetic field field at a point P (see the attached picture for Question 4) which is equidistant from the capacitor plates, denoted by coordinates $(s,\phi,z=0)$, is given by

$$\frac{I}{2\pi a^2}s\hat{\phi}$$

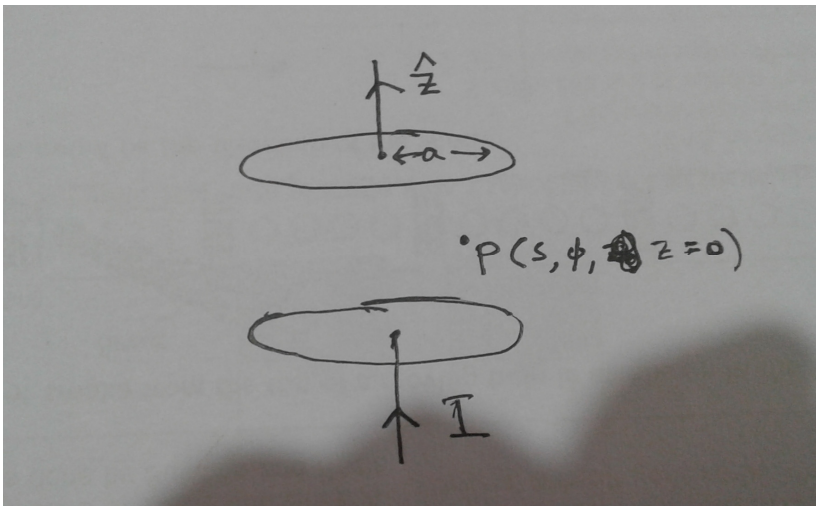
4c.) Calculate the Poynting vector $\vec{P} = \frac{\vec{E}x\vec{B}}{\mu_0}$ at the above point and explain its direction in physical terms.

Q5. [Total Marks: 10+4+6=20]

a.)Write the Maxwell's equations in a charge free space and show that they imply wave equations for the electric and magnetic fields.

b.) Suppose we take the plane wave electric field traveling in the z direction $\vec{E} = \vec{E}_o \cos(kz - \omega t)$ where $\vec{E}_o$ is constant vector as one possible solution of the wave equation. Prove that $\vec{E}_o$ must lie in the xy plane.

c.) Determine the magnetic field associated with the above electric field.



Picture for Question 4